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# WHEN DOUBTING TELLEGEN MATERIAL GIVE HER THE BENEFIT OF THE DOUBT

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**Abstract–** *In two recent articles in the* International Journal of Infrared and Millimeter Waves, *Lakhtakia* [1994a,b] *has questioned the soundness of the investigations of magnetoelectric materials, and of Tellegen media (nonreciprocal bi-isotropic media) in particular. This communication is an attempt to stand in defence of Tellegen.*

## Introduction

To describe the most general bianisotropic linear material, take the following constitutive relations:

$$\bar{D} = \overline{\overline{\epsilon}} \cdot \bar{E} + \overline{\overline{\xi}} \cdot \bar{H} \tag{1}$$

$$\bar{B} = \overline{\overline{\mu}} \cdot \bar{H} + \overline{\overline{\zeta}} \cdot \bar{E} \tag{2}$$

where the four material dyadics, permittivity $\overline{\overline{\epsilon}}$, permeability $\overline{\overline{\mu}}$, and the magnetoelectric ones, $\overline{\overline{\xi}}$ and $\overline{\overline{\zeta}}$, transmit the relation between the electric and magnetic field quantities $\bar{E}, \bar{H}$ and the flux quantities $\bar{D}, \bar{B}$.

Under this characterization we can find several materials that exhibit special electromagnetic effects: chiral, nonreciprocal, bi-isotropic, and gyrotropic are examples. These labels are popular in today's electromagnetics literature, and it is hardly necessary to emphasize that novel nonisotropic materials could, at least theoretically, be used for exciting applications in microwave and millimeter wave technology.

How general can these materials be? What are the limits of the media effects in terms of constitutive characterization? These questions have stirred controversy among students of bianisotropic and magnetoelectric materials.[1]

Since a full dyadic in the three-dimensional space possesses nine components, the general bianisotropic description includes 36 material parameters. Recently, it has been claimed that this description is loose since it allows too many degrees of freedom [LAKHTAKIA AND WEIGLHOFER 1994]. In particular, if we restrict the discussion to *bi-isotropic* media, where the four coefficients are scalars,[2]

$$\bar{D} = \epsilon\bar{E} + \xi\bar{H} \tag{3}$$

$$\bar{B} = \mu\bar{H} + \zeta\bar{E} \tag{4}$$

Then the claim of [LAKHTAKIA AND WEIGLHOFER 1994] stands as:

$$\xi + \zeta = 0 \tag{5}$$

To interpret (5), a useful way of rewriting (3)–(4) comes from [LINDELL ET AL. 1994] with the chirality parameter $\kappa$ and nonreciprocity parameter $\chi$,

$$\bar{D} = \epsilon\bar{E} + (\chi - j\kappa)\sqrt{\mu_0\epsilon_0}\bar{H} \tag{6}$$

$$\bar{B} = \mu\bar{H} + (\chi + j\kappa)\sqrt{\mu_0\epsilon_0}\bar{E} \tag{7}$$

where the constants $\mu_0, \epsilon_0$ have been extracted for dimensionality reasons, and $j$ emphasizes the frequency-domain nature of the relations[3]. Material

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[1] See, for example the recent electronic discussions in the list `chiral-l` of `listserv@listserv.earn.net`

[2] Sometimes the constitutive relations of the form (1)–(2) go under the name of *Tellegen.* Another important representation is the *Post* form, where $\bar{D}$ and $\bar{H}$ are expressed as functions of $\bar{E}$ and $\bar{B}$.

[3] The convention $\exp(j\omega t)$ is implied.

with $\kappa \neq 0$ has been called *Pasteur medium*, and that with $\chi \neq 0$ follows the label *Tellegen medium*.[4]

Now, condition (5) is easily seen to require $\chi = 0$, and the conclusion of [LAKHTAKIA AND WEIGLHOFER 1994] can be therefore paraphrased with the following sentence: "Tellegen media cannot exist."

Since I have elsewhere [SIHVOLA 1994] replied to the universality claim of the conjecture (5), I shall not dwell in its implications here. Rather, I would like to examine critically two other attacks against Tellegen media which have recently appeared on the pages of this journal [LAKHTAKIA 1994a,b].

[LAKHTAKIA 1994a] focuses on the properties of the constitutive relations that those investigators were using who first predicted and experimentally studied the nonreciprocity in magnetic substances. This castigation of 60's' magnetoelectric scholars was followed by the hilarious travesty about Tellegen medium being "a Boojum, you see" [LAKHTAKIA 1994b].

Let us see, then, what there is inside [LAKHTAKIA 1994a,b].

## Approximative constitutive relations

A considerable theoretical and experimental research effort was expended in the end of 1950's and early 1960's on the "magnetoelectric effect," with which term it is meant that in certain materials we may observe magnetic polarization as the sample is exposed to an electric field (ME$_E$ effect), or, correspondingly, an external magnetic field creates electric polarization into this medium (ME$_H$ effect).

**Early magnetoelectric studies–** The magnetoelectric effect depends on the crystal structure of the magnetic material classes[5] Following Landau and Lifshitz, Dzyaloshinskii [1960] was able to show that among the well-known antiferromagnetic substances there is one, namely chromium oxide, where the magnetoelectric effect should occur from symmetry considerations. Soon, Astrov [1961] experimentally confirmed this prediction, as he measured the finite magnetic moment of a sample of single-crystal

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[4] Since Tellegen medium is both nonreciprocal and bi-isotropic, it is sometimes called NRBI medium.

[5] See the comprehensive tables in the systematic text on symmetry and magnetism [BIRSS 1964].

chromium oxide, placed in an electric field. The measurements were carried out at the frequency of 10 kHz. O'Dell spent several years studying the theoretical base of the magnetoelectric effect, performed measurements of the effect as it can be observed in $Cr_2O_3$, and discussed the use of the effect for magnetic memories [O'DELL 1965a, 1965b, 1966, 1970].

The target of the critique in [LAKHTAKIA 1994a] is the way these people (mostly Dzyaloshinskii and O'Dell) use constitutive relations. The main objection is that these relations strictly apply only to fields with no temporal variation, and since static conditions do not exist, and no known material medium has instantaneous response, this starting point is nonphysical.

**Static analysis–** Indeed, since the early investigators were not microwave engineers looking for optical activity phenomena in materials but rather found interest in static and low-frequency fields, their constitutive relations also reflected this starting point. More precisely, they used relations (1)–(2) where $\overline{\overline{\xi}} = \overline{\overline{\zeta}}$. Before connecting this condition with time variation, let us make a note of the usefulness of static field analysis.

Static field solutions often can be successfully used in quasistatic and dynamic problems. Consider, for example, the calculation of attenuation due to rain. Even at the microwave frequency of 3 GHz one can use the static solution to calculate the internal field and dipole moment of a raindrop because the size of the drop is much smaller than the wavelength of the radio wave. In the same vein, also the bianisotropic relations with assumptions of frozen time-dependence are valid up to certain frequencies of the field variation. Sure, one can raise the reasonable question whether the frequencies of the measurement are small enough for the quasistatic assumption to be valid. The normal frequency ranges of the magnetoelectric measurements are around kilohertz region. The corresponding wavelength is so large that definitely no significant spatial variation in the field amplitude can be observed across the sample cross section. If the quantitativity of this comparison does not sound convincing, one may refer to Figures 4a,b of [CLIN ET AL. 1990] where the static and dynamic[6] magnetoelectric effects of cobalt-iodine boracite have been measured, and the curves are almost indistuingishable.

**Dispersion in magnetoelectric parameters–** The frequency dependence

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[6] Using an a.c. magnetic field of amplitude 10 Oe (1 mT), and frequency of 160 Hz.

of the magnetoelectric parameters is actually different for the chirality parameter $\kappa$ and the nonreciprocity parameter $\chi$. Consider the "Drude–Born–Fedorov" representation of reciprocal chiral media [Lakhtakia et al. 1989]

$$\bar{D} = \epsilon(\bar{E} + \beta\nabla \times \bar{E}) \tag{8}$$

$$\bar{B} = \mu(\bar{H} + \beta\nabla \times \bar{H}) \tag{9}$$

where $\beta$ is again the chirality parameter, proportional to $\kappa$ [Sihvola and Lindell 1991]. This representation shows that, since in statics the field is curl-free, chirality does not have effect. The low-frequency dependence of the chirality parameter $\kappa$ is proportional to frequency, vanishing at zero frequency. On the other hand, the nonreciprocity parameter $\chi$ can be observed in statics since it contributes to the thermodynamic potential, linear in both the electric and magnetic field which can be independent [Landau and Lifshitz 1984, p. 176].

**Exclusion of RBI–** The second investigative note raised by Lakhtakia against the constitutive relations is their negation of RBI media. RBI refers here to reciprocally bi-isotropic materials, especially chiral, optically active media.[7] Lakhtakia's statement is true: setting $\overline{\overline{\xi}} = \overline{\overline{\zeta}}$ in the constitutive relations forces $\kappa = 0$ in the bi-isotropic case, as is easily seen. This is actually a consequence of the static assumption, whilst the electromagnetic chirality effect only becomes visible as the frequency is high enough.

But this fact was clearly articulated in the magnetoelectric analyses. O'Dell explicitly states his assumptions of "*nongyrotropic*, linear, lossless, homogeneous, and time-independent media" [O'Dell 1970, p. 43]. Gyrotropy exhibits itself in the rotation of the polarization plane, like in optically active materials.[8] Hence nongyrotropic assumption also excludes chirality. Little wonder then that the O'Dell and Dzyaloshinskii formulations can be logically manipulated towards contradiction with RBI. They were outruled in the first place.

**Incomplete measurements?–** Another objection Lakhtakia raises against the early experimentalists is the fact that they only measured one of the

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[7] Media with $\kappa \neq 0, \chi = 0$.

[8] It may be worth noting that sometimes the terms "chirality" and "gyrotropy" are reserved to separate effects [Barron 1982, p. 127]. But often optical activity is a subset of gyrotropic phenomena, especially in the former–USSR literature (see, for example, [Landau and Lifshitz 1984, p. 348]).

magnetoelectric factors, *i.e.*, either $\overline{\overline{\xi}}$ or $\overline{\overline{\zeta}}$ in relations (1)–(2) but not both. The natural reason for this has been the fact that different measurement setups were required for electrically induced and magnetically induced magnetoelectric effects. But in fact it is not needed. This is since here the closeness to statics — so heavily denounced by Lakhtakia — is a virtue: the lower the frequency of the operating system, the more accurate is the nonreciprocity effect in the measured magnetoelectric phenomenon since chirality effect vanishes in statics (*cf.* relations (6)–(7)).

Finally, O'Dell's actual measurements [1966] on both magnetoelectric cross-components — although subject to measurement uncertainties — present us evidence on the fact that the magnetoelectric effect is due to nonreciprocity ($\xi$ and $\zeta$ of the same sign) and not to chirality ($\xi$ and $\zeta$ of the opposite sign). O'Dell's measurements yield the following results for these two parameters: $-2.20 \cdot 10^{-4}$ $(\pm 10\%)$ and $-1.85 \cdot 10^{-4}$ $(+20\%, -10\%)$. This speaks quite strongly for the fact that that the measured effect is due to nonreciprocity rather than chirality[9].

**Outcome from [LAKHTAKIA 1994a]–** In conclusion of this reinvestigation of the early studies of the magnetoelectric effect, I claim that the then-used constitutive relations have been consistent both in statics and at those low frequencies of the 1960's. Furthermore, references to RBI media are not relevant since optical activity effects are of second- order at these frequencies.[10]

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[9] Note that here we are talking about the axial component of the anisotropic magnetoelectric effect of chromium oxide.

[10] Regarding the final "troublesome issue" (Section 5d of [LAKHTAKIA 1994a]), namely the attenuation analysis; this problem arose from a sign error in an earlier study by O'Dell. The issue was cleared already in 1965 by [FUCHS], resulting in the consensus that "[p]lane waves can $\cdots$ propagate without attenuation for any value of the magnetoelectric coupling constant," and confirmed by O'Dell [1966].

## A Snark, a Boojum, or just NRBI?

A beautifully composed allegory as such, "The Tellegen medium is 'a Boojum, you see' " ([LAKHTAKIA 1994b]) searches metaphors for Tellegen medium. In this pursuit, Lakhtakia is more successful than the sailors seeking Snark: fiction provides him plenty of nonexistent objects to construct the story.[11] But what is the scientific message therein?

Summarizing [LAKHTAKIA 1994b] on the Tellegen medium, it is first shown that for homogeneous Tellegen material outside sources, the electric and magnetic fields behave just like in a medium where the Tellegen pseudoscalar vanishes.[12] Also, when fields are analyzed within electric source region, the NRBI material remains indistuingishable from the corresponding isotropic material. Only if we measure the fields within magnetic source region, we can feel the difference, but even there, equivalent electric sources can be constructed for which the difference vanishes.

The first response to these findings is to ask why fear equivalence. We have several examples of sources that are equivalent to each other and media for which the eigenwaves do not feel certain differences in the material parameters [LINDELL 1992, Chapter 6]. But perhaps a more conspicious weakness of [LAKHTAKIA 1994b] is the assumption of material homogeneity throughout the article. Indeed, if our universe were composed of homogeneous Tellegen medium, we would not be able to tell apart whether our medium is Tellegen medium or dielectric. Anyway, compared to the dullness of such a world, I would rate this is as a small problem.

But as soon as there are real objects, we have inhomogeneities and boundaries. Consequently there are reflections and scattering of the electromagnetic waves. Then, what does it mean "to see," which is what the title of [LAKHTAKIA 1994b] asks us to do?

Seeing an object is the act of receiving waves reflected from and scattered by it.[13] Symptomatic or not, it is especially in reflection that the Tellegen

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[11] Nonexistent? Listed are Boojum's five unmistakable marks [CARROLL 1876]: taste, the habit of getting up late, the slowness in taking a jest, the fondness of bathing-machines, and its ambition. Could a nonexistent being carry properties like these?

[12] Strictly speaking, this is valid for the Post constitutive relations. Using the relations (3)–(4), the situation is not that simple.

[13] For our Tellegen purposes, we may set aside the physiological and mental aspects of the process.

parameter $\chi$ (*cf*. relations (6)–(7)) has effect.[14] Therefore also reflection measurements are most suitable for the determination of the Tellegen parameter of NRBI media. See Chapter 7 of [LINDELL ET AL. 1994]. Boojums disappear from sight.

## Finally, of falsificationism and existence theorems

The text above has not proven the existence of the Tellegen medium. It is a trial to disprove some attempts to prove Tellegen medium nonexistent. The case is still open.

Science progresses through bold hypotheses and fierce efforts to refute these conjectures. The hypothesis cannot be confirmed by refering to failed attacks it has withstood, nor on experimental evidence which agrees with its predictions. We only can falsify wrong hypotheses. This is the idea behind Popper's falsificationism [1963].

It may look paradoxical to support both falsificationism and the possible existence of Tellegen media. (Existence theorems cannot be falsified.) But perhaps both are valid in their own domains. Along with this hope, be the present study dedicated to the memory of Karl R. Popper, who, last Saturday, passed away.

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[14] This can be complemented with the chirality parameter $\kappa$ which mostly affects the properties of the propagating wave.

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