Ubi materia, ibi geometria

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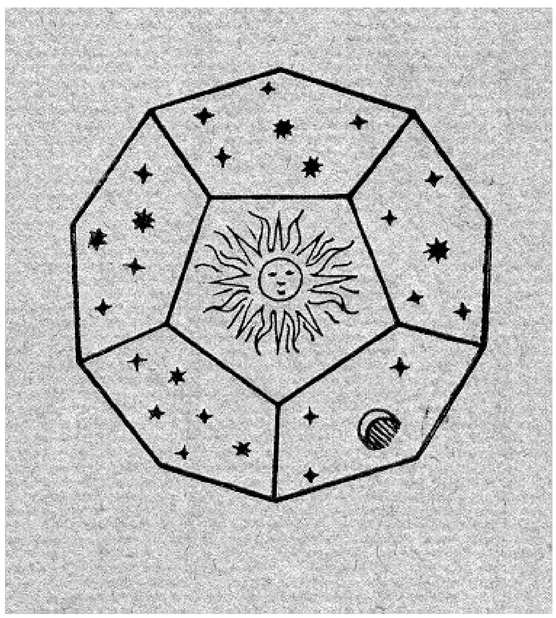


Abstract

This study discusses the connection between geometry and matter. The theme of the paper, ubi materia, ibi geometria, is illuminated from the point of view of post-Renaissance science and also in terms of modern bi-anisotropic electromagnetics research. The citation is pinpointed in Johannes Kepler's philosophical texts, and an attempt is made to understand its meaning. The validity of 'ubi materia, ibi geometria' for the electromagnetic bi-anisotropics research is reconfirmed and a modified interpretation is proposed for it.

This article is a revised and expanded version of the Report 339 [1] of the HUT Electromagnetics Laboratory Report Series. The report itself was a full-paper exposition of the invited presentation *Ubi materia, ibi geometria*, held at the *Bianisotropics 2000* meeting (8th International Conference on Electromagnetics of Complex media) in Lisbon, Portugal (27–29 September 2000) [2].

1. Preliminaries

Johannes Kepler declared 'Ubi materia, ibi geometria'. It is not difficult to imagine that a modern researcher in the field of chiral and bi-anisotropic electromagnetics concurs with this statement; 'where there is matter, there is geometry'. Matter and geometry, Nature and mathematics are intimately connected.

Ubi materia, ibi geometria. Citations to this formula appear often in scientific literature, sometimes with a reference to Kepler, sometimes just as it reads.[1] But it is difficult to find an exact citation from where and in what context in Kepler's writings this statement can be found. It seems to be well hidden; the only[2] place I have seen it is in the small tractate *De fundamentis astrologiae certioribus* [8]. The sentence is shown with the surrounding text in Figure 1. Where there is matter, there is geometry.

XX

Nos et variatas planetarum vires, et numerum, aliter, et forte non inconcinnius deducemus; quàm ARISTOTELES sua quatuor elementa ex quatuor qualitatum combinationibus. Variatio omnis à contrarietatibus est, et prima a primis. Primam contrarietatem ARISTOTELES in Metaphysicis recipit illam, quae est inter Idem et Aliud: volens supra Geometriam altius et generalius philosophari. Mihi Alteritas, in creatis nulla aliunde esse videtur, quàm ex materia, aut occasione materiae; at ubi materia, ibi Geometria. Itaque quam ARISTOTELES dixit primam contrarietatem sine medio, inter Idem et Aliud; eam ego in Geometricis, philosophicè consideratis, invenio esse primam quidem contrarietatem, sed cum medio, sic quidem, ut quod ARISTOTELI fuit ALIVD, unus terminus, eum nos in PLVS et MINVS, duos terminos dirimamus. Cum ergo Geometria toti mundo creando praebuerit exemplar, non ineptè haec Geometrica contrarietas ad ornatum mundi, qui consistit in variatis planetarum viribus, concurret.

Figure 1: Johannes Kepler's Thesis XX from *De fundamentis astrologiae certioribus*; (Opera Omnia, Vol. 1, p. 423 [8]; the appearance in this figure, however, is from the Gesammelte Werke (München, 1941). Also worth checking is the version in Koyré's critical text [11, p. 400, endnote 12]; cf. footnote 8 of the present article). —What is Kepler's intention in the fourth sentence? (The rectangular emphasis is mine, not in the original text.)

The observation of the connectedness of nature and mathematics was essential with the scientific revolution in the 16th and 17th centuries. In today's world where apparently nothing else

[1] *Ubi materia, ibi geometria* can be found as a motto for chapters in mathematics books (for example, [3]), in general studies of the birth of new physics and astronomy, and as contribution to obscure rhetorics in speculative quantum physics texts. In fact, not only mathematicians and physicists use this slogan. It can be found in other environments, like of course philosophy and history, but also surveying [4], linguistics [5], and even jurisprudence [6]. A search in the world-wide-web with the string <ubi materia, ibi geometria> returns several virtual sites in rather unexpected places, most of them confusing. In the hunt for the origins of the title of the present paper, more interesting hints can be found using the digitally stored scholarly journal archive JSTOR [7].

[2] With this I do not imply that Kepler may not have written it also elsewhere, for example in one of his great books (e.g., *Mysterium Cosmographicum, Astronomia Nova, Harmonices Mundi*).

than change is permanent, knowledge is said to be short-lived, and postmodernism is claimed to have replaced modernity.[3] But is it really so? Is Millennium the opening to a New Age? I would hesitate to agree. It is one of the aims of the present paper to defend Kepler's ethos which pronounces the reality of geometry in matter. If we are now going towards a new age, it should and could rather be a Renaissance of modernity.

2. Matter and geometry: Kepler's Platonism

Johannes Kepler (1571–1630) is rightfully seen as a central figure in the birth of the new physics. It is often said that the Copernican revolution[4] marks the end of Medieval times, and there indeed, Kepler's contribution was very essential in giving support to heliocentric cosmology and replacing the Aristotelian concepts in mechanics by new ones. For Kepler and Galileo, not to mention Newton, mathematics had to be included in the correct description of natural phenomena.

But in the case of Kepler the importance of mathematics was not only in arithmetic. Surely he was able to calculate tolerable approximations for the positions of planets. However, Kepler was in the sense Platonist that he believed in a rigorous and perfect structure of the universe. His celestial world had—at least in his early writings—a geometrically exact, nearly Pythagorean form, "his God [was] a geometer, and not an aritmetician" [11, p. 139]. This view is reflected in his models for the sizes of the planetary spheres that had harmonic ratios.[5] Kepler made ample use of various polyhedric volumes and it can be seen in the illustrations of his books, cf., e.g. Figure 2. The young Kepler was advised by Tycho Brahe to abandon his aprioristic speculations for more fruitful observational work. The incompatibility between the two approaches, the empiristic Tychonian on one hand, and geometrically ideal on the other, caused probably certain tension for Kepler during his later studies.

Aside from astronomical and astrological theories, also in his Earth-bound studies Kepler was seeking causes for the form of matter. There he can be seen to follow more faithfully the metaphysics of Aristoteles.[6] For example, in his study of the snowflake (*Strena seu de Nive sexangula* ([12]),[7] Kepler searches the reason and cause for the sixfold symmetry, obvious in snow crystals (see, for example, Figure 3). His conclusion is that the cause for the six-sided shape of a snowflake is a formative faculty already present in water in the liquid state and in vapour.

What, then, did Kepler mean by his *Ubi materia, ibi geometria*? The citation, Figure 1 — *Mihi Alteritas, in creatis nulla aliunde esse videtur, quam ex materia, aut occasione materiae;*

[3]Physicists at large tend to have another opinion; see, for example, the delightful [9].

[4]The term 'revolution' may have overtones that pertain to certain schools of the philosophy of science; by this choice of words I am not attaching myself to any of these, nor criticising them. The reader fluent in Finnish finds the discussion in [10] enlightening in respect how literally the metaphor of revolution can be taken in connection of Copernicanism.

[5]Kepler saw that there had to be six planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn) because only then the five regular polyhedra can be circumscribed between their spheres. In the earlier Ptolemaic system, there was place for seven planets (Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn); hence the Copernican system has to be valid (*Mysterium Cosmographicum*). (One can, however, with justification doubt that the system presented by Copernicus in his book *De Revolutionibus Orbium Coelestium* was simpler or aesthetically more pleasing than the theory in Ptolemy's *Almagest*.)

[6]According to the classical Aristotelian analysis, there are four causes for entities (things): material, formal, efficient, and final.

[7]See also [13, 14].

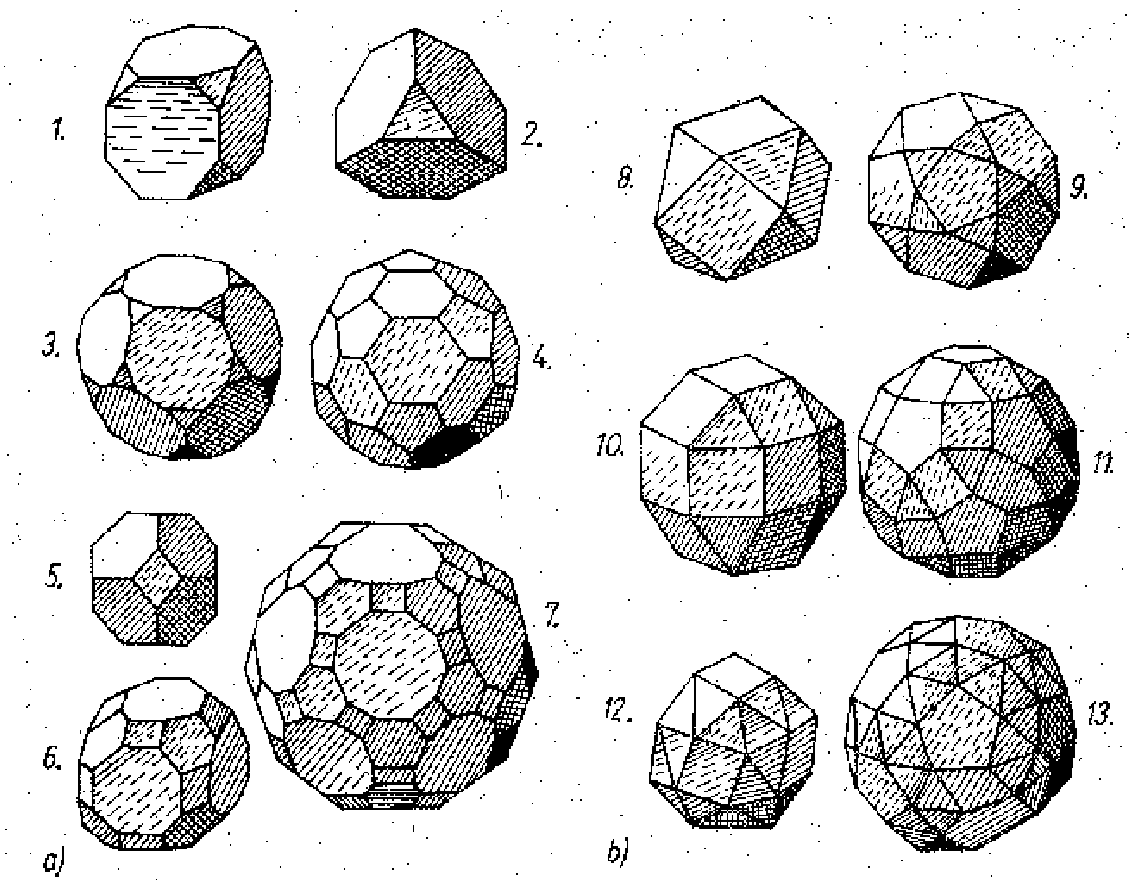


Figure 2: Archimedean polyhedra, used by Kepler in *Harmonice Mundi*.

at ubi materia, ibi geometria — is not perfectly apparent. Somehow Kepler is saying that matter, and its emergence (*occasione*) is the cause of a certain property in created objects: *alteritas*. 'Alteritas' might be 'variety' or 'otherness', and this is an essential concept which in some sense in Kepler's mind gets connected with geometry, as the conclusion of the sentence shows, in that it summarises the situation with 'where there is matter, there is geometry.'[8] One reasonable interpretation might be just to guess that Kepler emphasises his idealistic belief in perfect shapes immanent within all matter.[9] For example Koyré [11, p. 175] stresses the deep conviction of Kepler that there should not be any laxity between facts and theory.[10] Towards

[8]It is interesting to note that this focal point of the text reads differently here in the Opera Omnia edition of Kepler's works [8] (*aut occasione materiae,* at *ubi materia, ibi geometria*) than in another source, the study of Koyré [11, p. 400, endnote 12] (*aut occasione materiae,* aut *ubi materia, ibi geometria*). I feel an inclination to suspect that the version reproduced in Koyré's book is correct (*aut* instead of *at*), as it makes more sense to me. However, if the Opera Omnia text is more accurate, the interpretation of what Kepler tried to say has to be revised from the one presented in this section: *at* means "however"; *aut* could be translated here "in other words". Kepler's original text from 1602 would resolve this uncertainty.

[9]I welcome competing interpretations!

[10]Looking at the translation of the sentence in [17] with a materials scientist's viewpoint, one is left with many possibilities to understand Kepler's intention: *For me, the distinction among created things seems to be found in their matter, or on account on it; and where there is matter, there is geometry*. Little more help in reconstrucing Kepler's meaning can be gathered from the earlier translation by E. Meywald [18]: *To me it seems that diversity in things is created from nowhere other than matter, or from occasions caused by matter, and where there is matter there is geometry*. One further interpretation is the German translation [19]: *Mir scheint, ein "Anderssein" gibt es*

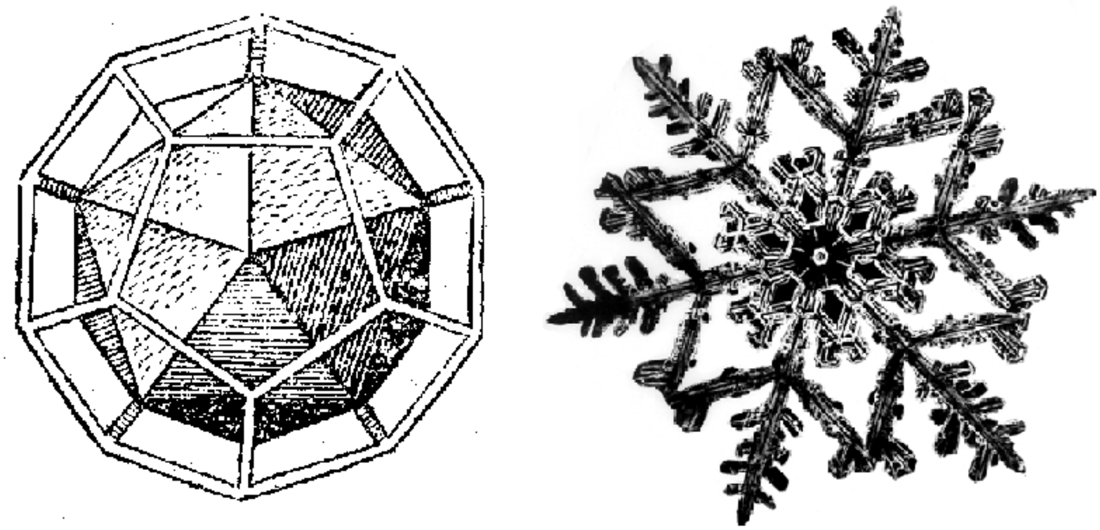


Figure 3: The togetherness of matter and geometry was obvious for Kepler also in his studies of snow [12]. On the left, one of his favourites, icosahedron in Platonic dodecahedron. To the right side, one of Wilson A. Bentley's snowflake photographs. (Spending decades during the early last century, W.A. Bentley of Jericho, Vermont (New England), patiently and skillfully took thousands of photographic images of snow crystals. These can be admired in the Dover edition [15] of his book and nowadays also on a CD-rom [16].) Note the sixfold symmetry in the snowflake.

the end of Thesis XX Kepler argues against Aristoteles, who juxtaposes the primary pair of opposites (*idem* and *aliud*; "same" and "different" as interpreted by Field) that have no middle term, and replaces this dichotomy by a gradual separation: "more" and "less" (*plus* and *minus*). This is the effect of geometry over philosophy. And therefore the variety and adornment in the world can be explained by geometrical principles as geometry is seen to allow more outcomes than Aristotelian logic. Quality differences are replaced by quantities: the properties are no longer cold or hot; moist or dry, but rather colder or hotter; moister or drier.[11]

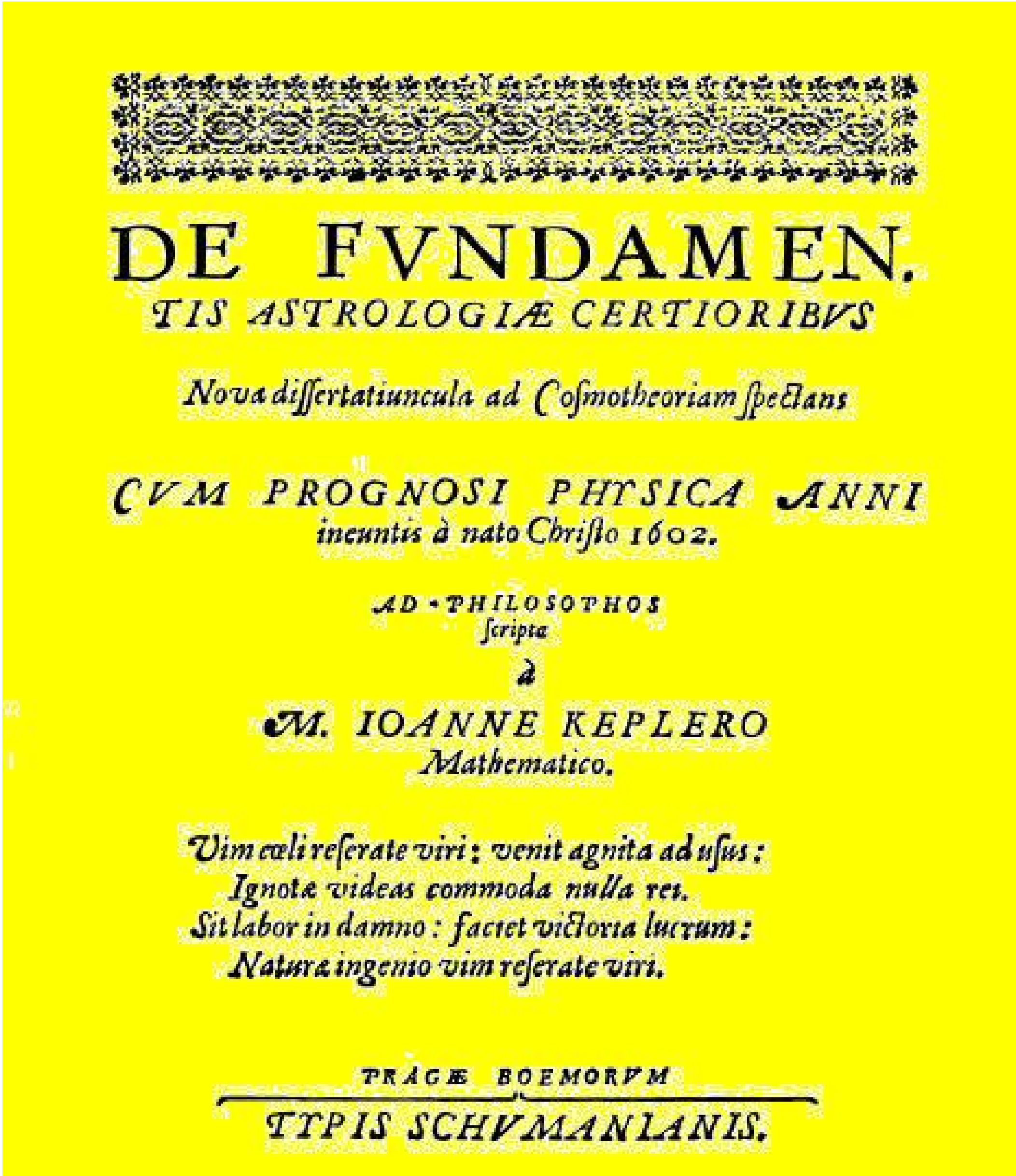
One may use another part of Kepler's text to support the above "Ubi materia, ibi geometria" -interpretation, namely thesis XL of the tractate. There the effects of the aspects of planets (special constellations corresponding to certain differences in longitudes) are discussed in terms of their effects of earthly phenomena. In Field's translation [17], " . . . every animal faculty is the image of God practising geometry in creation, and is roused to action by this celestial Geometry or Harmony of Aspects."[12] [13]

in der Schöpfung nur im Bereich der Materie bzw. in einem bestimmten Zustand der Materie, und wo Materie ist, da ist Geometrie.

[11] Cf. the closing sentence of Thesis XX: *Cum ergo Geometria toti mundo creando praebuerit exemplar, non ineptè haec Geometrica contrarietas ad ornatum mundi, qui consistit in variatis planetarum viribus, concurret.*

[12] In original, *Scilicet omnis animalis facultas est imago Dei γεωμετροῦντος in creatione, excitaturque ad opus suum, hac caelesti Aspectuum Geometria, seu Harmonia*. The Caspar–Hammer translation reads "Jedes seelische Vermögen ist ein Abbild Gottes, der bei der Erschaffung Geometrie verwirklicht, und wird zu seinem Werk durch jene himmlische Geometrie oder Harmonie der Aspekte angeregt."

[13] See also the summary of the *De Fundamentis Astrologiae Certioribus* by Max Caspar and Franz Hammer in the *Nachbericht* of the Part IV of the Kepler's collected works they edited [20].



DE FVNDAMEN.
TIS ASTROLOGIÆ CERTIORIBVS

Nova diſſertatiuncula ad Coſmotheoriam ſpectans

CVM PROGNOSI PHYSICA ANNI
ineuntis à nato Chriſto 1602.

AD PHILOSOPHOS
ſcripta
à
M. IOANNE KEPLERO
Mathematico.

Vim cœli reſerate viri: venit agnita ad uſus:
Ignotæ videas commoda nulla rei.
Sit labor in damno: faciet victoria lucrum:
Naturæ ingenio vim reſerate viri.

PRAGÆ BOEMORVM
TYPIS SCHVMANIANIS.

Figure 4: The title page of [8], taken from Gesammelte Werke (Beck).

Kepler wrote his study after the death of Tycho Brahe[14] in the final months of 1601, in a haste, according to Kepler scholars. It is interesting to take a look at the title page of [8], copied in Figure 4. There, in his tractate, despite the astrology-related title, Kepler was talking to philosophers as a mathematician, as the title page of his study shows. The kind of astrology Kepler was practicing had little to do with occultism. Field [17] discusses the distinction between high/low astrology, and also natural/judicial astrology, where in the former ones sublunary phenomena were of interest, like weather, mineralogy, and calendar making, and the latter ones contain soothsaying and the like. Kepler belongs to the high tradition of astrology.

The translation of the title page of [8] in Figure 4 has been given by J.V. Field in [17] as

[14]Kepler called Brahe The Phoenix of Astronomers [8, Thesis VI].

ON GIVING ASTROLOGY SOUNDER FOUNDATIONS
A new short dissertation concerning Cosmology
. . .
WITH A PHYSICAL PROGNOSIS FOR THE COMING YEAR
1602 after the birth of Christ.
Addressed to philosophers
by
Magister JOHANNES KEPLER
Mathematician.
. . .
Discover the force of the Heavens, O Men; once recognised it can be put to use.
That of which we are ignorant can profit us nothing.
Only futile labour is onerous: success brings gain.
By your skill, O Men, discover the force of Nature.
. . .
PRAGUE IN BOHEMIA

SCHUMANN PRESS

In addition to Kepler's own stress on his mathematical (instead of philosophical) background, very conspicuous on this title page is the emphasis on exploitation of nature. These lines display very much a particular profit-oriented view of knowledge and "success science" that is much criticised, even derided for its narrowness in our world of today. On the other hand, in Kepler's times such perspectives of utilisation of natural laws meant probably a qualitative broadening of the dominant (although certainly very holistic) world view.

Anyway, it seems that Kepler's praise for technical applicability of nature remains rhetorical. In his interest of earthly phenomena, his approach is mathematical, in particular geometric. Against this background of geometry and properties of materials it is probably not too bold to presume that Kepler would have loved today's solid-state physics classes, and would have much preferred them over semiconductor engineering textbooks where impurities and defects ruin the clean symmetries in solid matter.

3. Geometry and matter: Bi-anisotropics in the 21th century

There are several ways to look at the connection between geometry and matter in the framework of modern and 20th-century physics. An obvious and certainly important one of such approaches, which however is not the focus of this presentation, is the interaction of matter with not the microscopic geometry of material objects, but with the geometry of space and time, the spacetime. The theory of general relativity provides us the laws how the trajectories of moving objects are determined by the distribution of mass. "Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve." [21]. This idea of seeing movement as a result of curved spacetime modified by nearby and distant objects is philosophically different from the Newtonian scene of movements where the background space is infinite, time-independent, and Euclidian. It might be tempting to speculate how Kepler's way of looking at space and time, which was not yet affected by Newton's bias towards absolute space, could be related to a modern view of geometrodynamics and matter.

That is not my intention here. Instead I focus on a spatially more local treatment of the connectedness of matter and geometry, especially on the electromagnetic level.[15]

Ubi materia, ibi geometria. These words can a materials scientist safely speak out in the *Bianisotropics 2000* meeting in Lisbon. To calculate the effective dielectric and magnetic medium parameters of simple or complex materials, knowledge about their geometrical structure has to be made use of. The macroscopic properties of solid materials are connected to their microscopic crystallography. This aspect of Kepler's maxim has even been reworded more strongly in Neumann's principle: No asymmetry may be exhibited by any property of the crystal which is not possessed by the crystal itself [22]. In other words, the special characteristics of the geometrical constellation of solid matter "shine through" in the properties of its measurable properties. Pierre Curie has expressed the principle in the form 'C'est la dissymétrie, qui creé le phénomène' (It is asymmetry which creates the phenomenon).

Indeed, where a physicist or an electrical engineer needs to find out *ab initio* the effective properties of matter, the presence or absence of symmetry, that is geometry, has to be dealt with. But could Kepler's watchword be approached from another direction? What about understanding it in a way such that the role of geometry is stressed more; so that geometry begets matter? In such a reading of the principle, we in some sense surpass the Neumann–Curie interpretation. The properties of matter are determined by its geometrical description; this is then true but trivial: what is essential is that these properties can be very varied and unexpected.

Perhaps a more fitting rearticulation of Kepler would then be "Ubi geometria, ibi materia". On a higher level, something new arises from matter, something that admittedly is created and determined by the geometry but yet, at the same time, something that is not a simple reproduction of the group-theoretical symmetry properties of the crystal. But are we then not violating the Neumann principle? Probably not. We can safely accept that all observable macroscopic properties are reducable to the geometric and symmetry properties of the material, to avoid having to argue against Neumann. But the new phenomena "emerge" on a higher level. They are hidden to a beholder looking for an easy and straightforward connection between the microscopic geometrical and macroscopic material properties. How can it be possible? To become convinced, I suggest to think of engineering applications such as chiral media, photonic band gap structures, or liquid crystals.

These are applications of electromagnetic wave-material interaction, and before discussing their special features, recall the basic material relations of bi-anisotropic electromagnetics:

$$\mathbf{D} = \epsilon \bullet \mathbf{E} + \xi \bullet \mathbf{H} \tag{1}$$

$$\mathbf{B} = \zeta \bullet \mathbf{E} + \mu \bullet \mathbf{H} \tag{2}$$

which condense the constitutive properties of matter as the electromagnetic wave sees them. Here, the field vectors are denoted here by $\mathbf{E}$ (electric) and $\mathbf{H}$ (magnetic), and the responses in this representation are the flux densities $\mathbf{D}$ (electric) and $\mathbf{B}$ (magnetic). The relations between these vectors are given by the material parameters, permittivity ϵ, permeability μ, and the two magnetoelectric cross terms ξ and ζ.[16] The geometry of matter determines the character of the

[15]Aside from general relativity, there is a direct line from Kepler's geometrical planetary studies to quantum mechanics. The precession of the major axis of the elliptical orbit of a planet does not remain constant in space if there is another planet nearby. The precession was quantified in the 1930's by Werner Heisenberg, and was an important contribution in modern physics.

[16]I am not discussing here the question which one of the magnetic fields ($\mathbf{H}$ or $\mathbf{B}$) is primary and should appear

quantities ϵ, μ, ξ, and ζ, and the nature of the product marked by $\bullet$ in (1)–(2). For the simplest isotropic case, the product is a plain multiplication by a scalar. The other extreme is that all four medium "parameters" are full dyadics (alternatively, tensors of second rank) and then the number of degrees of freedom in the bi-anisotropic description of such matter is $4 \times 9 = 36$.[17]

If the medium is anisotropic, its material dyadics, for example the permittivity $\overline{\overline{\epsilon}}$, have a structure which reflects faithfully the internal geometry of the matter, according to the Neumann principle. But how do the magnetoelectric effects arise? Obviously through geometry.

Chirality is one reciprocal type of these magnetoelectric effects. Chirality can be said to be present in materials that have a handed microstructure. If the medium is predominantly right- or left-handed, macroscopic effects of chirality can be observed. One observable optical or electromagnetic effect of chiral media is their ability to rotate the polarisation plane of the incident wave, and on the level of dipole moments in the medium, the effect is electrically caused magnetic polarisation density and vice versa. But these effects take place because of the left–right symmetry is broken.[18]

Another affirmation for the strong interpretation of the Kepler principle is artificial magnetism. 'Natural' magnetism is present in certain materials, like iron, and in electromagnetics applications this property is taken into account by the $B - H$ relation. There, the origin of the magnetic properties is not important. It is not easy to talk about the geometrical cause of magnetic permeability in quantum mechanics. But because macroscopic electric loops act as magnetic moments, it is possible to synthetise a medium with magnetic permeability by embedding circular or curved conducting elements in a non-conducting matrix. Also in modelling of chiral, bi-anisotropic, and other complex materials, the observation of the appearance of magnetic properties for materials with non-magnetic constituents has been emphasised [28, 29, 30]. And not only for synthetic materials: if interested in "artificial" magnetism in natural media, see [31] for the diamagnetic effect of wet snow due to the funicularly circling water phase in the ice matrix.

Finally, it is perhaps proper to remind of one important electromagnetics application that hinges on the geometry–matter interaction. This is the field with many names: photonic band gaps, electromagnetic or photonic superlattices, photonic crystals, periodic quasicrystal structures, etc. [32]. There, the geometry of the structure is built in such a way that from the electromagnetic point of view the medium looks transparent or opaque depending on the wavelength, thus giving access to a whole new range of material properties. With such prospects in sight, it is not overoptimistic to foresee a revival for Kepler's credo. *Ubi materia, ibi geometria.*

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on the right-hand side of the constitutive relations (to read a sharp-worded argumentation against a sloppy use of different quantities and units for the magnetic field, look [23]). Also, I am well aware of the fact that one has to be careful with the order of the magnitude and balance of electric and magnetic polarisation terms on the right-hand side of these relations (see, for example, [24]). In terms of the focus of the paper, these points are not crucial; also, the form of the relations (1)–(2) is familiar and much-used in Bianisotropics meetings [25].

[17] This number is outrageously high. Brave attempts to control its freedom have found their way into the electromagnetics literature [26].

[18] In the subatomic level, physicists use the term 'parity' in connection of chirality. In the weak interaction process, parity is broken [27].

for the observation that I am trying to reconstruct in footnote 15. Officers in both the Electrical Engineering Department Library and the Main Library in the Helsinki University of Technology assisted me very much in locating old and rare texts. I also appreciate the comments of many colleagues which were made during and after my talk on this topic in the Bianisotropics 2000 meeting [2].

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