S-parametrien normalisointi-impedanssien muuttaminen

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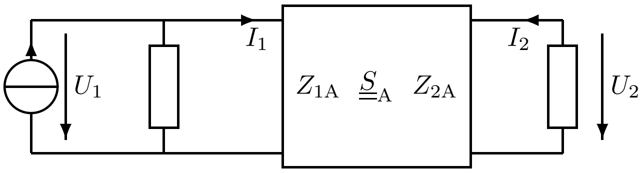
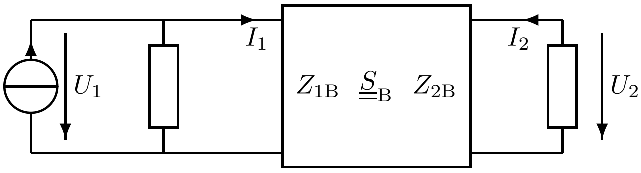
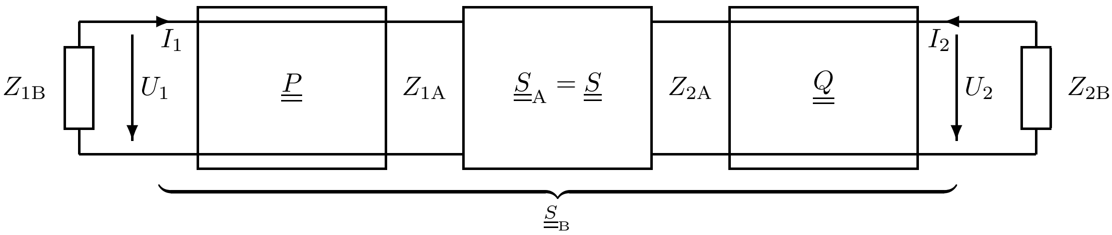
Seuraavassa sirontaparametrit $\underline{\underline{S}}_\mathrm{A}$ on normalisoitu impedansseihin $Z_{i\mathrm{A}}$ ($i = 1, 2, \ldots$) ja saman piirin parametrit $\underline{\underline{S}}_\mathrm{B}$ vastaavasti normalisointi-impedansseihin $Z_{i\mathrm{B}}$. Huom! Normalisointi-impedanssit oletetaan aina resistiivisiksi! Ne voivat olla eri suuret eri porteissa. S-parametrit kuvaavat laitteen toimintaa; niiden arvot eivät riipu siitä, mihin piiri on kytketty.





Nyt johdetaan muunnoskaava matriisien $\underline{\underline{S}}_\mathrm{A}$ ja $\underline{\underline{S}}_\mathrm{B}$ välille. Käytän kahta eri menetelmää, joista ensimmäinen perustuu suoran läpimenon sirontamatriisiin. Tämä näyttäisi olevan nopeampi tapa johtaa tarvittava kaava, mutta tuloksen laajentaminen moniporttiseen tapaukseen onnistuu paremmin myöhemmin käsiteltävällä "z-matriisin kautta kiertävällä" menetelmällä.

Muodostetaan kolmen sirontamatriisin ketju: suora läpimeno $\underline{\underline{P}}$ normalisointi-impedanssista $Z_{1\mathrm{B}}$ normalisointi-impedanssiin $Z_{1\mathrm{A}}$, lähtökohtana oleva sirontamatriisi $\underline{\underline{S}}_\mathrm{A}$ ja lopuksi suora läpimeno $\underline{\underline{Q}}$ impedanssista $Z_{2\mathrm{A}}$ impedanssiin $Z_{2\mathrm{B}}$:



Elementti P_{11} lasketaan heijastuskertoimesta ja elementti P_{21} on siirtotehovahvistuksen neliöjuuri; reaaliluvuilla laskut ovat helppoja. Fysikaalisesti tilanne vastaa kahden pitkän siirtojohdon liitoskohtaa.

$$P_{11} = -P_{22} = \frac{Z_{1\mathrm{B}} - Z_{1\mathrm{A}}}{Z_{1\mathrm{A}} + Z_{1\mathrm{B}}} \tag{1}$$

$$P_{21} = P_{12} = \sqrt{\frac{\frac{(\tau U)^2}{Z_{1\mathrm{A}}}}{\frac{U^2}{Z_{1\mathrm{B}}}}} = \sqrt{\frac{Z_{1\mathrm{B}}}{Z_{1\mathrm{A}}} \underbrace{\left(\frac{2Z_{1\mathrm{A}}}{Z_{1\mathrm{A}} + Z_{1\mathrm{B}}}\right)^2}_{\tau^2}} \tag{2}$$

$$P = \begin{bmatrix} \frac{Z_{1A}-Z_{1B}}{Z_{1A}+Z_{1B}} & \frac{2\sqrt{Z_{1A}Z_{1B}}}{Z_{1A}+Z_{1B}} \\ \frac{2\sqrt{Z_{1A}Z_{1B}}}{Z_{1A}+Z_{1B}} & \frac{Z_{1B}-Z_{1A}}{Z_{1A}+Z_{1B}} \end{bmatrix} = \frac{1}{Z_{1A}+Z_{1B}} \begin{bmatrix} Z_{1A}-Z_{1B} & 2\sqrt{Z_{1A}Z_{1B}} \\ 2\sqrt{Z_{1A}Z_{1B}} & Z_{1B}-Z_{1A} \end{bmatrix} \quad (3)$$

$$Q = \begin{bmatrix} \frac{Z_{2B}-Z_{2A}}{Z_{2A}+Z_{2B}} & \frac{2\sqrt{Z_{2A}Z_{2B}}}{Z_{2A}+Z_{2B}} \\ \frac{2\sqrt{Z_{2A}Z_{2B}}}{Z_{2A}+Z_{2B}} & \frac{Z_{2A}-Z_{2B}}{Z_{2A}+Z_{2B}} \end{bmatrix} = \frac{1}{Z_{2A}+Z_{2B}} \begin{bmatrix} Z_{2B}-Z_{2A} & 2\sqrt{Z_{2A}Z_{2B}} \\ 2\sqrt{Z_{2A}Z_{2B}} & Z_{2A}-Z_{2B} \end{bmatrix} \quad (4)$$

Näistä muodostetaan kirjan kaavoilla ensin $\underline{\underline{P}}$:n ja $\underline{\underline{S}}_A$:n peräkkäinkytkentä $\underline{\underline{R}}$. S-parametreistä jätetään yksinkertaisuuden vuoksi alaindeksi A pois:

$$R = \begin{bmatrix} \frac{P_{11}-\Delta P S_{11}}{1-P_{22}S_{11}} & \frac{P_{12}S_{12}}{1-P_{22}S_{11}} \\ \frac{P_{21}S_{21}}{1-P_{22}S_{11}} & \frac{S_{22}-\Delta S P_{22}}{1-P_{22}S_{11}} \end{bmatrix} = \frac{1}{1-P_{22}S_{11}} \begin{bmatrix} P_{11}-\Delta P S_{11} & P_{12}S_{12} \\ P_{21}S_{21} & S_{22}-\Delta S P_{22} \end{bmatrix}$$

Lasketaan tarvittavat determinantit valmiiksi:

$$\Delta R = R_{11}R_{22} - R_{12}R_{21} = \frac{(P_{11}-\Delta P S_{11})(S_{22}-\Delta S P_{22}) - P_{12}P_{21}S_{12}S_{21}}{(1-P_{22}S_{11})^2} \quad (6)$$

$$= \frac{P_{11}S_{22} - \Delta P \Delta S}{1-P_{22}S_{11}} \quad (7)$$

$$\Delta P = \frac{\overbrace{(Z_{1A}-Z_{1B})(Z_{1B}-Z_{1A})}^{2Z_{1A}Z_{1B}-Z_{1A}^2-Z_{1B}^2} - 4Z_{1A}Z_{1B}}{(Z_{1A}+Z_{1B})^2} = \frac{-(2Z_{1A}Z_{1B}+Z_{1A}^2+Z_{1B}^2)}{(Z_{1A}+Z_{1B})^2} = -1$$

$$\Rightarrow \Delta Q = -1 \quad (9)$$

Nyt voidaan muodostaa tavoitteena oleva matriisi $\underline{\underline{S}}_B$:

$$S_B = \frac{1}{1-R_{22}Q_{11}} \begin{bmatrix} R_{11}-\Delta R Q_{11} & R_{12}Q_{12} \\ R_{21}Q_{21} & Q_{22}-\Delta Q R_{22} \end{bmatrix} \quad (10)$$

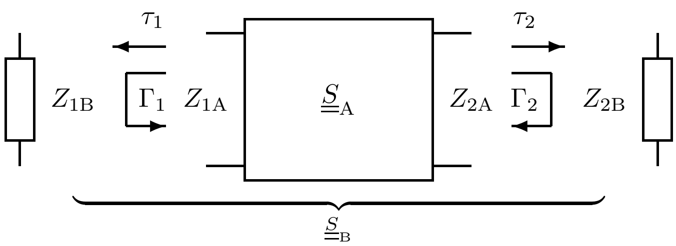
$$= \frac{1}{1-\frac{S_{22}-\Delta S P_{22}}{1-P_{22}S_{11}}Q_{11}} \begin{bmatrix} \frac{P_{11}-\Delta P S_{11}}{1-P_{22}S_{11}} - \frac{P_{11}S_{22}-\Delta P\Delta S}{1-P_{22}S_{11}}Q_{11} & \frac{P_{12}S_{12}}{1-P_{22}S_{11}}Q_{12} \\ \frac{P_{21}S_{21}}{1-P_{22}S_{11}}Q_{21} & Q_{22}-\Delta Q\frac{S_{22}-\Delta S P_{22}}{1-P_{22}S_{11}} \end{bmatrix}$$

$$S_B = \frac{1}{1-P_{22}S_{11}-(S_{22}-\Delta S P_{22})\,Q_{11}} \cdot$$
$$\begin{bmatrix} P_{11}+S_{11}-(P_{11}S_{22}+\Delta S)\,Q_{11} & P_{12}S_{12}\,Q_{12} \\ P_{21}S_{21}\,Q_{21} & Q_{22}(1-P_{22}S_{11})+(S_{22}-\Delta S P_{22}) \end{bmatrix} \quad (11)$$

$$S_B = \frac{1}{1-\frac{Z_{1B}-Z_{1A}}{Z_{1A}+Z_{1B}}S_{11} - S_{22}\frac{Z_{2B}-Z_{2A}}{Z_{2A}+Z_{2B}} + \Delta S\frac{Z_{1B}-Z_{1A}}{Z_{1A}+Z_{1B}}\frac{Z_{2B}-Z_{2A}}{Z_{2A}+Z_{2B}}} \cdot$$
$$\begin{bmatrix} \frac{Z_{1A}-Z_{1B}}{Z_{1A}+Z_{1B}} + S_{11} - \frac{Z_{1A}-Z_{1B}}{Z_{1A}+Z_{1B}}\frac{Z_{2B}-Z_{2A}}{Z_{2A}+Z_{2B}}S_{22} - \Delta S\frac{Z_{2B}-Z_{2A}}{Z_{2A}+Z_{2B}} & \frac{2\sqrt{Z_{1A}Z_{1B}}}{Z_{1A}+Z_{1B}}\frac{2\sqrt{Z_{2A}Z_{2B}}}{Z_{2A}+Z_{2B}}S_{12} \\ \frac{2\sqrt{Z_{1A}Z_{1B}}}{Z_{1A}+Z_{1B}}\frac{2\sqrt{Z_{2A}Z_{2B}}}{Z_{2A}+Z_{2B}}S_{21} & \frac{Z_{2A}-Z_{2B}}{Z_{2A}+Z_{2B}} - \frac{Z_{1B}-Z_{1A}}{Z_{1A}+Z_{1B}}\frac{Z_{2A}-Z_{2B}}{Z_{2A}+Z_{2B}}S_{11} + S_{22} - \Delta S\frac{Z_{1B}-Z_{1A}}{Z_{1A}+Z_{1B}} \end{bmatrix} \quad (12)$$

$$S_{\mathrm{B}} = \begin{bmatrix} S_{11\mathrm{B}} & S_{12\mathrm{B}} \\ S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \frac{S_{11\mathrm{A}}+\Gamma_1\Gamma_2 S_{22\mathrm{A}}-\Gamma_2\Delta S_{\mathrm{A}}-\Gamma_1}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_{\mathrm{A}}} & \frac{\tau_1\tau_2\sqrt{\frac{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}{Z_{1\mathrm{B}}Z_{2\mathrm{B}}}}\,S_{12\mathrm{A}}}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_{\mathrm{A}}} \\ \frac{\tau_1\tau_2\sqrt{\frac{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}{Z_{1\mathrm{B}}Z_{2\mathrm{B}}}}\,S_{21\mathrm{A}}}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_{\mathrm{A}}} & \frac{S_{22\mathrm{A}}+\Gamma_1\Gamma_2 S_{11\mathrm{A}}-\Gamma_1\Delta S_{\mathrm{A}}-\Gamma_2}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_{\mathrm{A}}} \end{bmatrix}$$

$$\Gamma_i = \frac{Z_{i\mathrm{B}} - Z_{i\mathrm{A}}}{Z_{i\mathrm{B}} + Z_{i\mathrm{A}}} \qquad \tau_i = \frac{2Z_{i\mathrm{B}}}{Z_{i\mathrm{A}} + Z_{i\mathrm{B}}} \tag{14}$$



Yllä on valmis muunnoskaava kaksiporttiselle tapaukselle. Seuraavassa tutkitaan samaa tilannetta lähtien S-parametrien määrittely-yhtälöistä. Menetelmä perustuu toisessa PDF-tiedostossa esitettyyn S- ja z-parametrien väliseen muunnokseen, josta on lyhennelmä seuraavassa:

$$\begin{bmatrix} \frac{U_1-Z_1I_1}{2\sqrt{Z_1}} \\ \frac{U_2-Z_2I_2}{2\sqrt{Z_2}} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \frac{U_1+Z_1I_1}{2\sqrt{Z_1}} \\ \frac{U_2+Z_2I_2}{2\sqrt{Z_2}} \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} U_1 - Z_1I_1 \\ U_2 - Z_2I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{Z_1} & 0 \\ 0 & \sqrt{Z_2} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{Z_1}} & 0 \\ 0 & \frac{1}{\sqrt{Z_2}} \end{bmatrix}}_{\underline{\underline{A}}} \begin{bmatrix} U_1 + Z_1I_1 \\ U_2 + Z_2I_2 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \underbrace{(\underline{\underline{I}} - \underline{\underline{A}})^{-1}(\underline{\underline{I}} + \underline{\underline{A}}) \underbrace{\begin{bmatrix} Z_{1\mathrm{A}} & 0 \\ 0 & Z_{2\mathrm{A}} \end{bmatrix}}_{\underline{\underline{Z}}_{\mathrm{A}}}}_{\underline{\underline{z}}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{17}$$

Piirin z-matriisi on vakio eikä siis riipu S-parametrien normalisointi-impedansseista. z-parametrit ovat siis seuraavat:

$$z_{\mathrm{A}} = (I - A)^{-1}(I + A)Z_{\mathrm{A}} \tag{18}$$

$$z_{\mathrm{B}} = (I - B)^{-1}(I + B)Z_{\mathrm{B}} = z_{\mathrm{A}} \tag{19}$$

$$B = \begin{bmatrix} \sqrt{Z_{1\mathrm{B}}} & 0 \\ 0 & \sqrt{Z_{2\mathrm{B}}} \end{bmatrix} \begin{bmatrix} S_{11\mathrm{B}} & S_{12\mathrm{B}} \\ S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{Z_{1\mathrm{B}}}} & 0 \\ 0 & \frac{1}{\sqrt{Z_{2\mathrm{B}}}} \end{bmatrix} \tag{20}$$

Seuraavassa jätän matriisimerkit laiskuuttani pois:

$$(I-A)^{-1}(I+A)Z_\mathrm{A} = (I-B)^{-1}(I+B)Z_\mathrm{B} \tag{21}$$

$$(I-B)(I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1} = (I+B) \tag{22}$$

$$(I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1} - B(I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1} = I+B \tag{23}$$

$$(I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1} - I = B(I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1} + B \tag{24}$$

$$B = \left(\underbrace{(I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1}}_{P} - I\right)\left(\underbrace{(I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1}}_{P} + I\right)^{-1} \tag{25}$$

$$P = (I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1} \tag{26}$$

Tarkastellaan ensin vertailun vuoksi kaksiporttista tapausta; lopussa on yhteenvetona moniporttinen sovellus. B on hieman eri muodossa kuin aiemmin, mutta molemmat esitystavat ovat identtiset:

$$\underbrace{\begin{bmatrix} S_{11\mathrm{B}} & \sqrt{\frac{Z_{1\mathrm{B}}}{Z_{2\mathrm{B}}}}S_{12\mathrm{B}} \\ \sqrt{\frac{Z_{2\mathrm{B}}}{Z_{1\mathrm{B}}}}S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{2\mathrm{B}}}{Z_{1\mathrm{B}}}} \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} S_{11\mathrm{B}} & S_{12\mathrm{B}} \\ S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix}}_{S_\mathrm{B}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{1\mathrm{B}}}{Z_{2\mathrm{B}}}} \end{bmatrix}}_{b}$$

Ratkaistaan matriisi S_B ja tehdään tarvittavat sijoitukset, jotta tulosta voidaan verrata edellä olleeseen muunnoskaavaan. Tästä alkaa armoton pyörittely — voit halutessasi siirtyä suoraan tiedoston loppuun. LaTeXilla on helppoa kehitellä pitkiä lausekkeita "*copy-paste*":n ja "*search-and-replace*"*n* avulla.

$$\begin{bmatrix} S_{11\mathrm{B}} & S_{12\mathrm{B}} \\ S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{1\mathrm{B}}}{Z_{2\mathrm{B}}}} \end{bmatrix}}_{a^{-1}} \underbrace{(P-I)\,(P+I)^{-1}}_{B} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{2\mathrm{B}}}{Z_{1\mathrm{B}}}} \end{bmatrix}}_{b^{-1}} \tag{28}$$

$$\underbrace{\begin{bmatrix} S_{11\mathrm{A}} & \sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}} \\ \sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}} & S_{22\mathrm{A}} \end{bmatrix}}_{A} = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}} \end{bmatrix} \begin{bmatrix} S_{11\mathrm{A}} & S_{12\mathrm{A}} \\ S_{21\mathrm{A}} & S_{22\mathrm{A}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}} \end{bmatrix}$$

Lasketaan edellisen perusteella matriisi P:

$$P = \begin{bmatrix} 1-S_{11\mathrm{A}} & -\sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}} \\ -\sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}} & 1-S_{22\mathrm{A}} \end{bmatrix}^{-1} \begin{bmatrix} 1+S_{11\mathrm{A}} & \sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}} \\ \sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}} & 1+S_{22\mathrm{A}} \end{bmatrix} Z_\mathrm{A}Z_\mathrm{B}^{-1}$$

$$\Delta = (1-S_{11\mathrm{A}})(1-S_{22\mathrm{A}}) - S_{12\mathrm{A}}S_{21\mathrm{A}} = 1 - S_{11\mathrm{A}} - S_{22\mathrm{A}} + \Delta S \tag{31}$$

$$P = \frac{1}{\Delta}\begin{bmatrix} 1-S_{22\mathrm{A}} & \sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}} \\ \sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}} & 1-S_{11\mathrm{A}} \end{bmatrix} \begin{bmatrix} 1+S_{11\mathrm{A}} & \sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}} \\ \sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}} & 1+S_{22\mathrm{A}} \end{bmatrix} Z_\mathrm{A}Z_\mathrm{B}^{-1} \tag{32}$$

$$= \frac{1}{\Delta}\left[\begin{matrix} (1-S_{22\mathrm{A}})(1+S_{11\mathrm{A}}) + S_{12\mathrm{A}}S_{21\mathrm{A}} \\ \sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}}(1+S_{11\mathrm{A}}) + (1-S_{11\mathrm{A}})\sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}} \end{matrix} \right.$$

$$\left.\begin{matrix} (1-S_{22\mathrm{A}})\sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}} + \sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}}(1+S_{22\mathrm{A}}) \\ S_{12\mathrm{A}}S_{21\mathrm{A}} + (1-S_{11\mathrm{A}})(1+S_{22\mathrm{A}}) \end{matrix}\right] Z_{\mathrm{A}}Z_{\mathrm{B}}^{-1} \tag{33}$$

$$= \frac{1}{\Delta}\begin{bmatrix} 1-S_{22\mathrm{A}}+S_{11\mathrm{A}}-S_{11\mathrm{A}}S_{22\mathrm{A}}+S_{12\mathrm{A}}S_{21\mathrm{A}} & 2\sqrt{\frac{Z_{1\mathrm{A}}}{Z_{2\mathrm{A}}}}S_{12\mathrm{A}} \\ 2\sqrt{\frac{Z_{2\mathrm{A}}}{Z_{1\mathrm{A}}}}S_{21\mathrm{A}} & S_{12\mathrm{A}}S_{21\mathrm{A}}-S_{11\mathrm{A}}S_{22\mathrm{A}}+1-S_{11\mathrm{A}}+S_{22\mathrm{A}} \end{bmatrix} Z_{\mathrm{A}}Z_{\mathrm{B}}^{-1} \tag{34}$$

$$= \frac{1}{\Delta}\begin{bmatrix} (1-S_{22\mathrm{A}}+S_{11\mathrm{A}}-\Delta S)Z_{1\mathrm{A}} & 2\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}S_{12\mathrm{A}} \\ 2\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}S_{21\mathrm{A}} & (1-\Delta S-S_{11\mathrm{A}}+S_{22\mathrm{A}})Z_{2\mathrm{A}} \end{bmatrix} Z_{\mathrm{B}}^{-1} \tag{35}$$

$$= \frac{1}{\Delta}\begin{bmatrix} (1-S_{22\mathrm{A}}+S_{11\mathrm{A}}-\Delta S)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} & 2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & (1-\Delta S-S_{11\mathrm{A}}+S_{22\mathrm{A}})\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} \end{bmatrix} \tag{36}$$

B:n lausekkeessa tarvitaan seuraavia välituloksia:

$$(P+I)^{-1} \tag{37}$$

$$= \Delta\begin{bmatrix} (1+S_{11\mathrm{A}}-S_{22\mathrm{A}}-\Delta S)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}+\Delta & 2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & (1-S_{11\mathrm{A}}+S_{22\mathrm{A}}-\Delta S)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}+\Delta \end{bmatrix}^{-1}$$

$$= \frac{\Delta}{\Delta_2}\begin{bmatrix} (1-S_{11\mathrm{A}}+S_{22\mathrm{A}}-\Delta S)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}+\Delta & -2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ -2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & (1+S_{11\mathrm{A}}-S_{22\mathrm{A}}-\Delta S)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}+\Delta \end{bmatrix}$$

$$(P-I) \tag{38}$$

$$= \frac{1}{\Delta}\begin{bmatrix} (1+S_{11\mathrm{A}}-S_{22\mathrm{A}}-\Delta S)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}-\Delta & 2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & (1-S_{11\mathrm{A}}+S_{22\mathrm{A}}-\Delta S)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}-\Delta \end{bmatrix}$$

Merkitään lyhyemmin: $F = 1-\Delta S$ ja $G = S_{11\mathrm{A}} - S_{22\mathrm{A}}$:

$$\Delta_2 = \left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}+\Delta\right)\left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}+\Delta\right) - 4S_{12}S_{21}\frac{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}{Z_{1\mathrm{B}}Z_{2\mathrm{B}}} \tag{39}$$

$$= \left(F^2-G^2-4S_{12}S_{21}\right)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + \Delta\left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}+(F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)+\Delta^2$$

Otetaan lausekkeen alkuosa käsittelyyn:

$$F^2-G^2-4S_{12}S_{21} = (1-\Delta S)^2-(S_{11}-S_{22})^2-4S_{12}S_{21} \tag{41}$$

$$= 1-2\Delta S+\Delta S^2-S_{11}^2-S_{22}^2+\underbrace{2S_{11}S_{22}-4S_{12}S_{21}}_{2\Delta S-2S_{12}S_{21}} \tag{42}$$

$$= \underbrace{1+\Delta S^2}\underbrace{-S_{11}^2-S_{22}^2-2\underbrace{S_{12}S_{21}}_{S_{11}S_{22}-\Delta S}} \tag{43}$$

Kokeillaan seuraavaa esitysmuotoa, jotta Δ voidaan ottaa yhteiseksi tekijäksi:

$$F^2-G^2-4S_{12}S_{21} = (1+a+\Delta S)\underbrace{(1-S_{11}-S_{22}+\Delta S)}_{\Delta} \tag{44}$$

$$= (1+a) - (1+a)S_{11} - (1+a)S_{22} + (1+a)\Delta S + \Delta S(1 - S_{11} - S_{22} + \Delta S)$$

$$= \underbrace{1 + \Delta S^2} + \underbrace{a - (1+a)S_{11} - (1+a)S_{22} + (1+a)\Delta S + \Delta S(1 - S_{11} - S_{22})}$$

Tämän pitäisi siis olla sama kuin aiemman lausekkeen:

$$-S_{11}^2 - S_{22}^2 - 2S_{11}S_{22} + 2\Delta S = a - (1+a)S_{11} - (1+a)S_{22} + (1+a)\Delta S + \Delta S(1 - S_{11} - S_{22})$$

$$-S_{11}^2 - S_{22}^2 - 2S_{11}S_{22} + S_{11} + S_{22} - \Delta S(-S_{11} - S_{22}) = a - aS_{11} - aS_{22} + a\Delta S \quad (48)$$

$$(S_{11} + S_{22}) - (S_{11} + S_{22})^2 + (S_{11} + S_{22})\Delta S = a - a(S_{11} + S_{22}) + a\Delta S \quad (49)$$

$$\Rightarrow a = S_{11} + S_{22} \quad (50)$$

$$\Delta_2 = (1 + \underbrace{S_{11} + S_{22}}_{a} + \Delta S)\Delta \frac{Z_{1A}}{Z_{1B}} \frac{Z_{2A}}{Z_{2B}} + \Delta \left((F+G)\frac{Z_{1A}}{Z_{1B}} + (F-G)\frac{Z_{2A}}{Z_{2B}} \right) + \Delta^2 \quad (51)$$

$$= \Delta \Bigg((1 + S_{11} + S_{22} + \Delta S)\frac{Z_{1A}}{Z_{1B}} \frac{Z_{2A}}{Z_{2B}} + (1 - \Delta S + S_{11} - S_{22})\frac{Z_{1A}}{Z_{1B}} +$$

$$(1 - \Delta S - S_{11} + S_{22})\frac{Z_{2A}}{Z_{2B}} + (1 - S_{11} - S_{22} + \Delta S) \Bigg) \quad (52)$$

$$\frac{\Delta_2}{\Delta} = \left(\frac{Z_{1A}}{Z_{1B}} \frac{Z_{2A}}{Z_{2B}} + \frac{Z_{1A}}{Z_{1B}} + \frac{Z_{2A}}{Z_{2B}} + 1 \right) + \left(\frac{Z_{1A}}{Z_{1B}} \frac{Z_{2A}}{Z_{2B}} + \frac{Z_{1A}}{Z_{1B}} - \frac{Z_{2A}}{Z_{2B}} - 1 \right) S_{11}$$

$$+ \left(\frac{Z_{1A}}{Z_{1B}} \frac{Z_{2A}}{Z_{2B}} - \frac{Z_{1A}}{Z_{1B}} + \frac{Z_{2A}}{Z_{2B}} - 1 \right) S_{22} + \left(\frac{Z_{1A}}{Z_{1B}} \frac{Z_{2A}}{Z_{2B}} - \frac{Z_{1A}}{Z_{1B}} - \frac{Z_{2A}}{Z_{2B}} + 1 \right) \Delta S \quad (53)$$

$$= \underbrace{\left(1 + \frac{Z_{1A}}{Z_{1B}}\right)}_{1+x} \underbrace{\left(1 + \frac{Z_{2A}}{Z_{2B}}\right)}_{1+y} + \left(-1 + \frac{Z_{1A}}{Z_{1B}}\right)\left(1 + \frac{Z_{2A}}{Z_{2B}}\right) S_{11}$$

$$+ \left(1 + \frac{Z_{1A}}{Z_{1B}}\right)\left(-1 + \frac{Z_{2A}}{Z_{2B}}\right) S_{22} + \left(1 - \frac{Z_{1A}}{Z_{1B}}\right)\left(1 - \frac{Z_{2A}}{Z_{2B}}\right) \Delta S \quad (54)$$

$$= (1+x)(1+y) + (-1+x)(1+y)S_{11} + (1+x)(-1+y)S_{22} + (1-x)(1-y)\Delta S \quad (55)$$

$$= (1+x)(1+y)\, \frac{(1+x)(1+y) + (-1+x)(1+y)S_{11} + (1+x)(-1+y)S_{22} + (1-x)(1-y)\Delta S}{(1+x)(1+y)}$$

$$= (1+x)(1+y) \left(1 + \frac{-1+x}{1+x} S_{11} + \frac{-1+y}{1+y} S_{22} + \frac{(1-x)(1-y)}{(1+x)(1+y)} \Delta S \right) \quad (56)$$

$$\frac{\Delta_2}{\Delta} = (1+x)(1+y)\,(1 - \Gamma_1 S_{11} - \Gamma_2 S_{22} + \Gamma_1 \Gamma_2 \Delta S) \quad (57)$$

Lasketaan lopuksi B ja sen avulla S_B:

$$B = \frac{1}{\Delta_2} \cdot \begin{bmatrix} (1 + S_{11A} - S_{22A} - \Delta S)\frac{Z_{1A}}{Z_{1B}} - \Delta & 2S_{12A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{2B}} \\ 2S_{21A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{1B}} & (1 - S_{11A} + S_{22A} - \Delta S)\frac{Z_{2A}}{Z_{2B}} - \Delta \end{bmatrix} \cdot$$

$$\begin{bmatrix} (1 - S_{11A} + S_{22A} - \Delta S)\frac{Z_{2A}}{Z_{2B}} + \Delta & -2S_{12A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{2B}} \\ -2S_{21A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{1B}} & (1 + S_{11A} - S_{22A} - \Delta S)\frac{Z_{1A}}{Z_{1B}} + \Delta \end{bmatrix} \quad (58)$$

$$= \frac{1}{\Delta_2} \cdot \begin{bmatrix} (F+G)\frac{Z_{1A}}{Z_{1B}} - \Delta & 2S_{12A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{2B}} \\ 2S_{21A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{1B}} & (F-G)\frac{Z_{2A}}{Z_{2B}} - \Delta \end{bmatrix} \cdot \begin{bmatrix} (F-G)\frac{Z_{2A}}{Z_{2B}} + \Delta & -2S_{12A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{2B}} \\ -2S_{21A}\frac{\sqrt{Z_{1A}Z_{2A}}}{Z_{1B}} & (F+G)\frac{Z_{1A}}{Z_{1B}} + \Delta \end{bmatrix} \quad (59)$$

$$
= \frac{1}{\Delta_2} \cdot \begin{bmatrix} \left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}+\Delta\right)\left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}-\Delta\right) - 2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}}2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & \left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}+\Delta\right)2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} - 2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}}\left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}-\Delta\right) \\ \left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}+\Delta\right)2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} - 2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}}\left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}-\Delta\right) & \left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}+\Delta\right)\left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}-\Delta\right) - 2S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}}2S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \end{bmatrix} \tag{60}
$$

$$
= \frac{1}{\Delta_2} \cdot \begin{bmatrix} \left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}+\Delta\right)\left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}-\Delta\right) - 4S_{12\mathrm{A}}S_{21\mathrm{A}}\frac{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}{Z_{1\mathrm{B}}Z_{2\mathrm{B}}} & 4\Delta S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 4\Delta S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & \left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}+\Delta\right)\left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}-\Delta\right) - 4S_{21\mathrm{A}}S_{12\mathrm{A}}\frac{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}{Z_{1\mathrm{B}}Z_{2\mathrm{B}}} \end{bmatrix} \tag{61}
$$

$$
= \frac{1}{\Delta_2} \cdot \begin{bmatrix} (F^2-G^2-4S_{12}S_{21})\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + \Delta\left((F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} - (F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right) - \Delta^2 & 4\Delta S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 4\Delta S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & (F^2-G^2-4S_{12}S_{21})\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + \Delta\left((F-G)\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - (F+G)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right) - \Delta^2 \end{bmatrix} \tag{62}
$$

$$
= \frac{\Delta}{\Delta_2} \cdot \begin{bmatrix} \begin{matrix}(1+S_{11}+S_{22}+\Delta S)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + (1-\Delta S+S_{11}-S_{22})\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} \\ -(1-\Delta S-S_{11}+S_{22})\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - (1-S_{11}-S_{22}+\Delta S)\end{matrix} & 4S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 4S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & \begin{matrix}(1+S_{11}+S_{22}+\Delta S)\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - (1-\Delta S+S_{11}-S_{22})\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} \\ +(1-\Delta S-S_{11}+S_{22})\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - (1-S_{11}-S_{22}+\Delta S)\end{matrix} \end{bmatrix} \tag{63}
$$

$$
B = \frac{\Delta}{\Delta_2} \cdot \begin{bmatrix} \begin{matrix}\left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} - \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - 1\right) + \left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} + \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + 1\right)S_{11} \\ + \left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} - \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + 1\right)S_{22} + \left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} + \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - 1\right)\Delta S\end{matrix} & 4S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 4S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & \begin{matrix}\left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} + \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - 1\right) + \left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} - \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + 1\right)S_{11} \\ + \left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} + \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + 1\right)S_{22} + \left(\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} + \frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}} - \frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}} - 1\right)\Delta S\end{matrix} \end{bmatrix} \tag{64}
$$

$$
= \frac{\Delta}{\Delta_2} \cdot \begin{bmatrix} \begin{matrix}\left(-1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right) + \left(1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)S_{11} \\ + \left(-1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(-1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)S_{22} + \left(1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(-1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)\Delta S\end{matrix} & 4S_{12\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 4S_{21\mathrm{A}}\frac{\sqrt{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & \begin{matrix}\left(-1-\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(1-\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right) + \left(-1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(-1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)S_{11} \\ + \left(1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)S_{22} + \left(-1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)\Delta S\end{matrix} \end{bmatrix} \tag{65}
$$

$$= \frac{\Delta}{\Delta_2} \cdot \begin{bmatrix} (-1+x)(1+y) + (1+x)(1+y)\,S_{11} + (-1+x)(-1+y)\,S_{22} + (1+x)(-1+y)\,\Delta S & 4S_{12\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}} \\ 4S_{21\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}} & (-1-x)(1-y) + (-1+x)(-1+y)\,S_{11} + (1+x)(1+y)\,S_{22} + (-1+x)(1+y)\,\Delta S \end{bmatrix} \tag{66}$$

$$= \frac{(1+x)(1+y)\Delta}{\Delta_2} \cdot \begin{bmatrix} \frac{(-1+x)(1+y)}{(1+x)(1+y)} + \frac{(1+x)(1+y)}{(1+x)(1+y)} S_{11} + \frac{(-1+x)(-1+y)}{(1+x)(1+y)} S_{22} + \frac{(1+x)(-1+y)}{(1+x)(1+y)} \Delta S & 4S_{12\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}(1+x)(1+y)} \\ 4S_{21\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}(1+x)(1+y)} & \frac{(-1-x)(1-y)}{(1+x)(1+y)} + \frac{(-1+x)(-1+y)}{(1+x)(1+y)} S_{11} + \frac{(1+x)(1+y)}{(1+x)(1+y)} S_{22} + \frac{(-1+x)(1+y)}{(1+x)(1+y)} \Delta S \end{bmatrix} \tag{67}$$

$$= \frac{(1+x)(1+y)}{(1+x)(1+y)\left(1 - \Gamma_1 S_{11} - \Gamma_2 S_{22} + \Gamma_1 \Gamma_2 \Delta S\right)} \cdot$$

$$\begin{bmatrix} \frac{(-1+x)}{(1+x)} + S_{11} + \frac{(-1+x)(-1+y)}{(1+x)(1+y)} S_{22} + \frac{(-1+y)}{(1+y)} \Delta S & 4S_{12\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}\left(1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)} \\ 4S_{21\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}\left(1+\frac{Z_{1\mathrm{A}}}{Z_{1\mathrm{B}}}\right)\left(1+\frac{Z_{2\mathrm{A}}}{Z_{2\mathrm{B}}}\right)} & -\frac{(1-y)}{(1+y)} + \frac{(-1+x)(-1+y)}{(1+x)(1+y)} S_{11} + S_{22} + \frac{(-1+x)}{(1+x)} \Delta S \end{bmatrix}$$

$$= \frac{1}{\left(1 - \Gamma_1 S_{11} - \Gamma_2 S_{22} + \Gamma_1 \Gamma_2 \Delta S\right)} \cdot$$

$$\begin{bmatrix} -\Gamma_1 + S_{11} + \Gamma_1\Gamma_2 S_{22} - \Gamma_2 \Delta S & 4S_{12\mathrm{A}} \frac{Z_{1\mathrm{B}} Z_{2\mathrm{B}} \sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}(Z_{1\mathrm{B}}+Z_{1\mathrm{A}})(Z_{2\mathrm{B}}+Z_{2\mathrm{A}})} \\ 4S_{21\mathrm{A}} \frac{Z_{1\mathrm{B}} Z_{2\mathrm{B}} \sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}(Z_{1\mathrm{B}}+Z_{1\mathrm{A}})(Z_{2\mathrm{B}}+Z_{2\mathrm{A}})} & -\Gamma_2 + \Gamma_1\Gamma_2 S_{11} + S_{22} - \Gamma_1 \Delta S \end{bmatrix} \tag{69}$$

Tästä saadaan helposti matriisi S_B:

$$\begin{bmatrix} S_{11\mathrm{B}} & S_{12\mathrm{B}} \\ S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{1\mathrm{B}}}{Z_{2\mathrm{B}}}} \end{bmatrix} \underbrace{(P-I)\,(P+I)^{-1}}_{B} \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\frac{Z_{2\mathrm{B}}}{Z_{1\mathrm{B}}}} \end{bmatrix} \tag{70}$$

$$= \frac{1}{\left(1 - \Gamma_1 S_{11} - \Gamma_2 S_{22} + \Gamma_1 \Gamma_2 \Delta S\right)} \cdot$$

$$\begin{bmatrix} -\Gamma_1 + S_{11} + \Gamma_1\Gamma_2 S_{22} - \Gamma_2 \Delta S & \sqrt{\frac{Z_{2\mathrm{B}}}{Z_{1\mathrm{B}}}} 4S_{12\mathrm{A}} \frac{Z_{1\mathrm{B}} Z_{2\mathrm{B}} \sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{2\mathrm{B}}(Z_{1\mathrm{B}}+Z_{1\mathrm{A}})(Z_{2\mathrm{B}}+Z_{2\mathrm{A}})} \\ \sqrt{\frac{Z_{1\mathrm{B}}}{Z_{2\mathrm{B}}}} 4S_{21\mathrm{A}} \frac{Z_{1\mathrm{B}} Z_{2\mathrm{B}} \sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{Z_{1\mathrm{B}}(Z_{1\mathrm{B}}+Z_{1\mathrm{A}})(Z_{2\mathrm{B}}+Z_{2\mathrm{A}})} & \sqrt{\frac{Z_{1\mathrm{B}}}{Z_{2\mathrm{B}}}} \sqrt{\frac{Z_{2\mathrm{B}}}{Z_{1\mathrm{B}}}} (-\Gamma_2 + \Gamma_1\Gamma_2 S_{11} + S_{22} - \Gamma_1 \Delta S) \end{bmatrix}$$

$$= \frac{1}{\left(1 - \Gamma_1 S_{11} - \Gamma_2 S_{22} + \Gamma_1 \Gamma_2 \Delta S\right)} \cdot$$

$$\begin{bmatrix} -\Gamma_1 + S_{11} + \Gamma_1\Gamma_2 S_{22} - \Gamma_2 \Delta S & 4S_{12\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{B}} Z_{2\mathrm{B}}} \sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{(Z_{1\mathrm{B}}+Z_{1\mathrm{A}})(Z_{2\mathrm{B}}+Z_{2\mathrm{A}})} \\ 4S_{21\mathrm{A}} \frac{\sqrt{Z_{1\mathrm{B}} Z_{2\mathrm{B}}} \sqrt{Z_{1\mathrm{A}} Z_{2\mathrm{A}}}}{(Z_{1\mathrm{B}}+Z_{1\mathrm{A}})(Z_{2\mathrm{B}}+Z_{2\mathrm{A}})} & -\Gamma_2 + \Gamma_1\Gamma_2 S_{11} + S_{22} - \Gamma_1 \Delta S \end{bmatrix} \tag{72}$$

Lopputulokset:

$$S_\mathrm{B} = \begin{bmatrix} S_{11\mathrm{B}} & S_{12\mathrm{B}} \\ S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \frac{S_{11\mathrm{A}}+\Gamma_1\Gamma_2 S_{22\mathrm{A}}-\Gamma_2\Delta S_\mathrm{A}-\Gamma_1}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_\mathrm{A}} & \frac{\tau_1\tau_2\sqrt{\frac{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}{Z_{1\mathrm{B}}Z_{2\mathrm{B}}}}S_{12\mathrm{A}}}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_\mathrm{A}} \\ \frac{\tau_1\tau_2\sqrt{\frac{Z_{1\mathrm{A}}Z_{2\mathrm{A}}}{Z_{1\mathrm{B}}Z_{2\mathrm{B}}}}S_{21\mathrm{A}}}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_\mathrm{A}} & \frac{S_{22\mathrm{A}}+\Gamma_1\Gamma_2 S_{11\mathrm{A}}-\Gamma_1\Delta S_\mathrm{A}-\Gamma_2}{1-\Gamma_1 S_{11\mathrm{A}}-\Gamma_2 S_{22\mathrm{A}}+\Gamma_1\Gamma_2\Delta S_\mathrm{A}} \end{bmatrix}$$

$$\Gamma_i = \frac{Z_{i\mathrm{B}} - Z_{i\mathrm{A}}}{Z_{i\mathrm{B}} + Z_{i\mathrm{A}}} \qquad \tau_i = \frac{2Z_{i\mathrm{B}}}{Z_{i\mathrm{A}} + Z_{i\mathrm{B}}} \tag{74}$$

S-parametrien normalisointi muuttuu siis $Z_{i\mathrm{A}}$:sta $Z_{i\mathrm{B}}$:hen. Tulos on tietysti sama kuin aiemmin toisella tavalla johdettu muunnoskaava. Seuraavassa esitysmuodossa tulos lienee kuitenkin yleistettävissä myös tapauksiin $n > 2$:

$$S_\mathrm{B} = \begin{bmatrix} S_{11\mathrm{B}} & S_{12\mathrm{B}} \\ S_{21\mathrm{B}} & S_{22\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{Z_{1\mathrm{B}}}} & 0 \\ 0 & \frac{1}{\sqrt{Z_{2\mathrm{B}}}} \end{bmatrix} (P-I)\,(P+I)^{-1} \begin{bmatrix} \sqrt{Z_{1\mathrm{B}}} & 0 \\ 0 & \sqrt{Z_{2\mathrm{B}}} \end{bmatrix}$$

missä

$$P = (I-A)^{-1}(I+A)Z_\mathrm{A}Z_\mathrm{B}^{-1} \tag{76}$$

$$A = \begin{bmatrix} \sqrt{Z_{1\mathrm{A}}} & 0 \\ 0 & \sqrt{Z_{2\mathrm{A}}} \end{bmatrix} \begin{bmatrix} S_{11\mathrm{A}} & S_{12\mathrm{A}} \\ S_{21\mathrm{A}} & S_{22\mathrm{A}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{Z_{1\mathrm{A}}}} & 0 \\ 0 & \frac{1}{\sqrt{Z_{2\mathrm{A}}}} \end{bmatrix} \tag{77}$$

$$Z_\mathrm{A} = \begin{bmatrix} Z_{1\mathrm{A}} & 0 \\ 0 & Z_{2\mathrm{A}} \end{bmatrix} \qquad Z_\mathrm{B} = \begin{bmatrix} Z_{1\mathrm{B}} & 0 \\ 0 & Z_{2\mathrm{B}} \end{bmatrix} \tag{78}$$